PAT Tree and PAT Array

Presented by Huiqin Körkel-Qu
Institute for Computer Linguistics, Heidelberg University

March 14, 2011

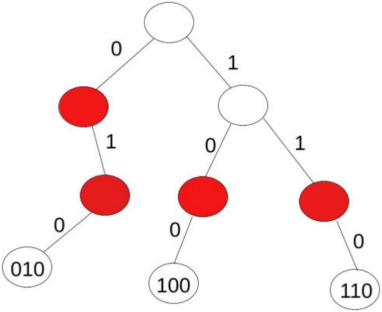
Outline

- Trie, patricia tree
- From semi-infinite strings to a PAT tree
- Algorithms on PAT tree
- Structures modified from a PAT tree

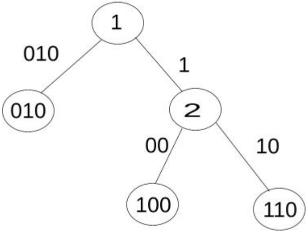
Trie and Patricia tree

- **trie**: originates from the word 're**trie**val'
  every path from root to a leave represents a string.
- **patricia tree:** space optimized trie
  nodes with only one child will be merged
- **example:** given strings 100, 010, 110

trie



patricia tree



Patricia tree

Patricia tree:

- every internal node has two children
- each internal node has an indication of branching
  1. **bit position to branch**
  2. 'zero' bit to left subtree
  3. 'one' bit to right subtree
- only useful (real) branching is produced!!

Semi-infinite strings (sistrings)

Given a text (character array), a **sistring**(semi-infinite string) of this text is:

a subsequence of this text
starting from some point (position) of the text
going until end of the text

Eg:

Text: this is an example of sistring....

sistring 1: this is an example of sistring....
sistring 2: his is an example of sistring...
sistring 9: an example of sistring...
sistring 13: xample of sistring...

order of the sistrings: $9 < 2 < 1 < 13$

Sistrings

Why sistring?

Humans can easily grasp all substrings of a text easily.

Eg:

text: 'HEID',
4 sistrings: 'HEID', 'EID', 'ID', 'D'
10 substrings: 'HEID', 'HEI', 'HE', 'H', 'EID', 'EI', 'E', 'ID', 'I', 'D'

saving only n sistrings, we can get all $n(n+1)/2$ substrings easily by prefix searching.

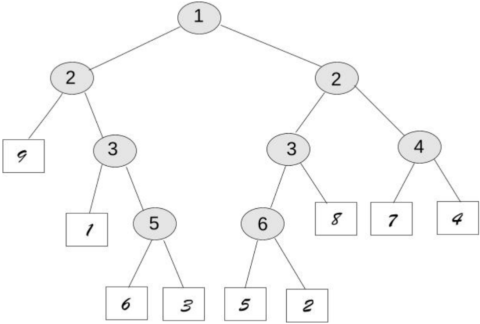
A PAT tree is a patricia tree over all sistrings of a text.

internal nodes: branching position
pointers to subtrees

external nodes: sistrings

Text: 01011011000111.....
Position: 123456789.....

sistring1: 01011011000111...
sistring2: 1011011000111...
sistring3: 011011000111...
sistring4: 11011000111...
sistring5: 1011000111...
sistring6: 011000111...
....

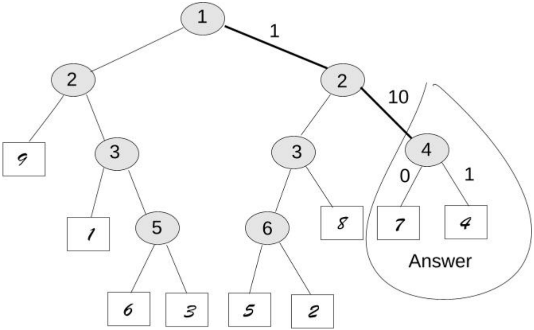


text size: n
tree size: $O(n)$ tree height: $O(\log n)$ to $O(n)$

Algorithms on PAT tree

- prefix searching
- proximity searching
- range searching
- longest repetition searching
- most frequent searching
- regular expression searching
- the longest palindrome searching

Eg: searching for the prefix 110



searching time: proportional to the query length
no more than the height of the tree

Proximity searching

Find all places where two strings (substrings in the text) are not too 'far away'

two strings: s_1 and s_2
distance: $b, b \in N$ (number of symbols, words, etc)

Eg:

$s_1 =$ 'cat', $s_2 =$ 'mouse', $b = 2$ (number of words)

'**cat** catches **mouse**' $\in s_1 b s_2$
'a **cat** has caught a **mouse**' $\notin s_1 b s_2$

Proximity searching

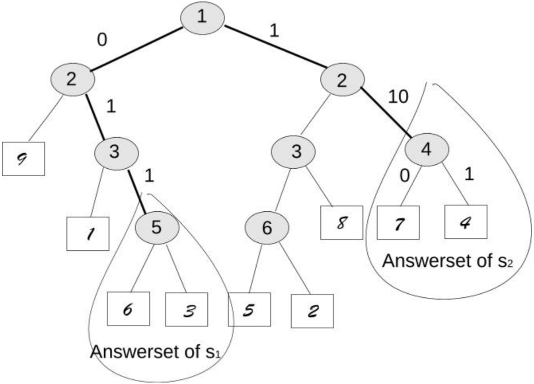
Proximity searching algorithm based on PAT tree:

- search for s_1 and s_2
- assume that the answer set sizes are m_1 and m_2 respectively and $m_1 \leq m_2$
- sort the answer of s_1 whose size is m_1
- check every answer in the answer set of s_2 to see if it satisfies the distance condition

complexity: sort + check $= m_1 \log m_1 + m_2 \log m_1$

Proximity searching

$s_1 = 011, s_2 = 110, b = 2$

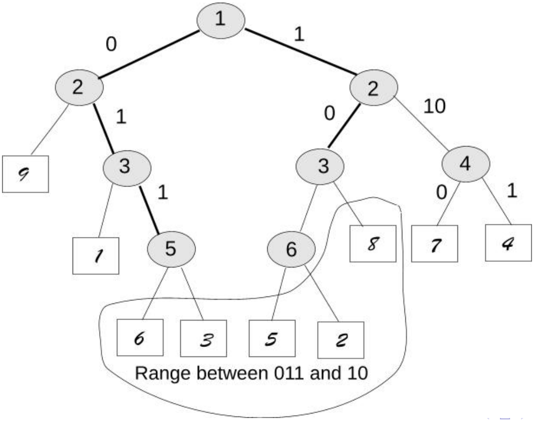


the final answer (no constraint on order): $\{(3,4),(6,7),(6,4)\}$

Range searching

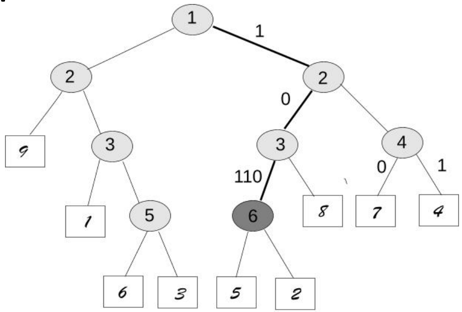
Eg: Searching in the lexicographical range 'ab' .... 'ad'
'abc' $\in$ range('ab', 'ad')
'aea' $\notin$ range('ab', 'ad')

searching in the range 011 and 10



Find the longest match between two different positions in a text. (the 'biggest' internal node)

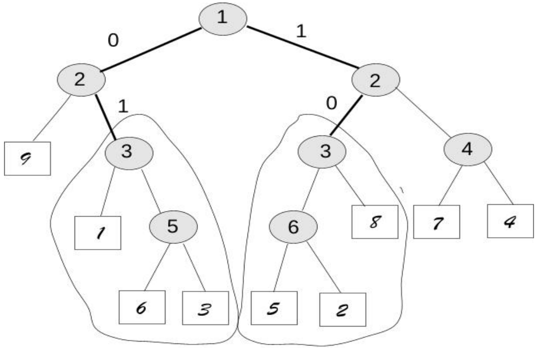
Example: 01011011000111



6 is the 'biggest' internal node
10110 is the longest repetition

Find the string that appears most frequently in the text.

Eg: find the most frequent substring of length 2
(the biggest subtree with distance 2 to root)



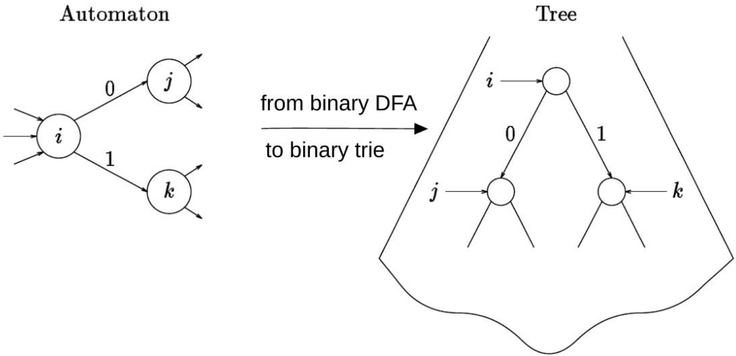
the 'biggest' subtrees at distance 2 from root node

The most frequent 2-grams are 01 and 10
both appear 3 times

Regular expression searching

- regular expression $\Rightarrow$ binary DFA (Deterministic Finite Automaton with input alphabet $\{0, 1\}$) without final state outgoing transition
- simulate the binary DFA on **binary trie**
  - initial state $\Rightarrow$ root
  - for transition $i \rightarrow_0 j$
    state $j \Rightarrow$ internal node (associated with state i)'s left child
  - for transition $i \rightarrow_1 j$
    state $j \Rightarrow$ internal node (associated with state i)'s right child
- if final state $\Rightarrow$ internal node, accept the whole subtree
- if final state $\Rightarrow$ external node, run DFA continue.

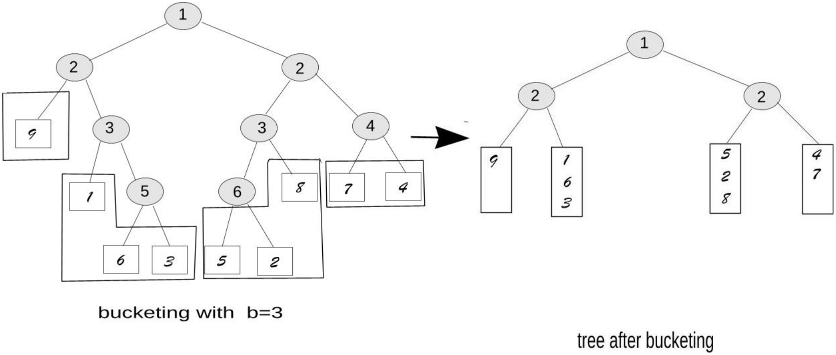
Regular expression searching



This figure is from Gonnet, Baeza-Yates and Snider (1992).

Bucketing the external nodes

Bucketing: replace subtrees (size limitation b) with buckets



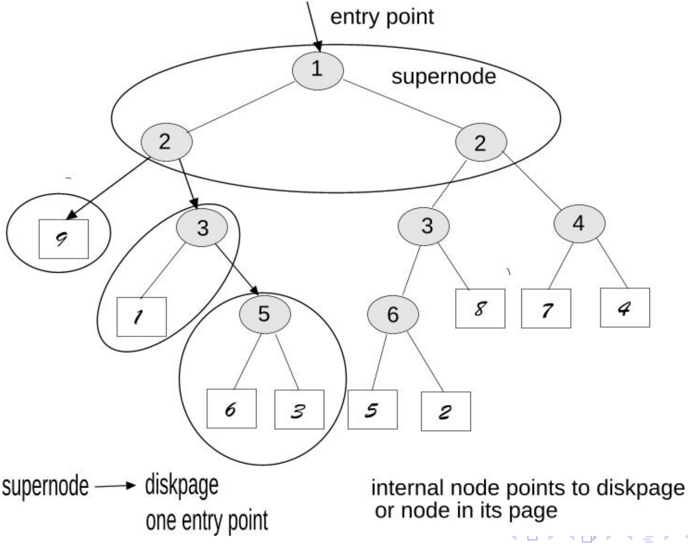
Some properties of bucketing

Bucketing: tradeoff between time and space

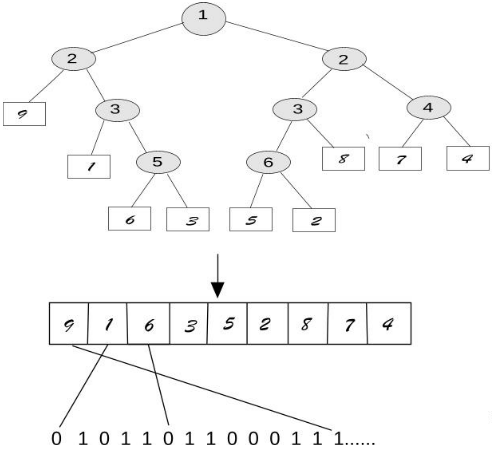
- not every bucket is full
- every bucket saves up to $b - 1$ internal nodes
- on average there are $b \ln 2$ keys per bucket
- for random text after bucketing there are $\frac{n}{b \ln 2}$ internal nodes in the tree left
- the searching time increases up to b

Supernodes–mapping tree on disk

Idea: big tree is stored in many disk pages
one page has only one entry



From PAT tree to PAT array



idea:

bucket the whole tree

keep the order of external nodes

construction:

quicksort

advantage:

keep most information of PAT tree

save space

disadvantage:

time complexity may increase

not all searching described above can be done

Construction of PAT array for large text

If a text is small, its PAT array can be built in memory.
what if the text is too big?

- cut the text into small pieces
- construct a PAT array for every piece in memory
- merge the PAT arrays

two merging cases

1. merge small array with large array
2. merge large arrays

Merge small with large arrays

What is stored in memory? (small and fast medium)

1. the small text
2. the array for small text
3. a **counter**

What is stored on hard disk? (large but slow storage medium)

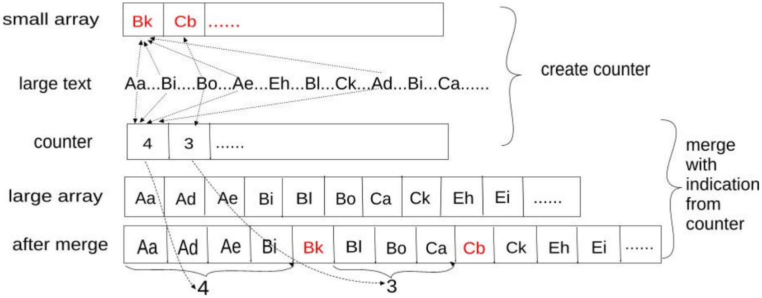
1. the large text
2. the array for the large text

What is the counter in memory?

- the counter contains an item for every sistring in the small array
- Item i in the counter indicates how many sistrings in the **large array** are between sistrings $(i-1)$ and i in the **small array**

Merge small with large arrays

The large text is read sequentially to create the **counter**



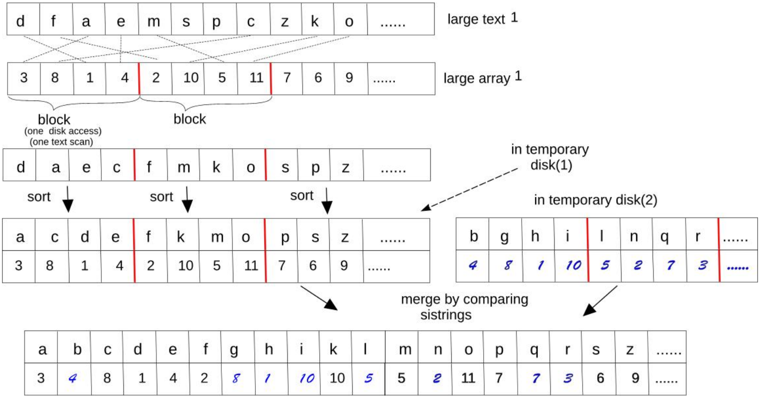
The sistrings in the small array are inserted into the large array according to the counter.

Merge large texts

Idea:

- reduce random access to hard disk
  read a block of pointers in PAT array instead of one by one
  Eg. If block size is m, and the text length is n, reading block by block needs $\lceil n/m \rceil$ times hard disk access with $\lceil n/m \rceil$ times text scan.
- sort sistrings of every block respectively
- put the results above into temporary disk space
- merge the PAT arrays by comparing sistrings (from two texts)

Merge large texts



Summary

PAT tree and PAT array are the data structures which

- preprocess text
- allow many different ways of searching
- fit for large text
- with high efficiency in space and time

Reference

- Gaston H. Gonnet, Ricardo A. Baeza-Yates, Tim Snider: New Indices for Text: PAT Trees and PAT arrays. In: Frakes, William; Baeza-Yates, Ricardo (eds.): Information Retrieval. Data Structures and Algorithms. Englewood Cliffs, N.J.: Prentice Hall, 1992
- Udi Manber, Ricardo A. Baeza-Yates: An algorithm for string matching with a sequence of don't cares. Information Processing Letters 37:133-136, 1991
- Ricardo A. Baeza-Yates, Gaston H. Gonnet: Fast text seaching for regular expressions or automaton searching on tries, Journal of the ACM (JACM), Volume 43 Issue 6, Nov. 1996
- Stefan Olk: PAT-Trees/PAT-Arrays. Ausarbeitung eines Vortrags für das Proseminar Online-Recherche Techniken im Wintersemester 1997/98

Reference

- Heart X.Raid: PAT Tree used in substring matching (In Chinese). http://www.javaeye.com/topic/615295
- Kenny Kwok: The Generic Chinese PAT Tree, Introduction to PAT-Tree and its variations. www.cse.cuhk.edu.hk/ lyu/student/mphil/kenny/PATTREE.ppt
- Bernd Mehnert: PAT-Trees. Seminarreferat 7.2.2005. http://kontext.fraunhofer.de/haenelt/kurs/Referate/Mehnert_WS05/ Trees-1.pdf